BLOQUE III: APLICACIONES AFINES

(21) a) $f(\overset{A}{0,0}) = (\overset{A'}{1,-1})$ $\quad \overrightarrow{AB} = (1,0)$ $\quad R = \{(0,0); (1,0), (0,1)\}$

$f(\overset{B}{1,0}) = (\overset{B'}{3,-1})$ $\quad \overrightarrow{AC} = (0,1)$

$f(\overset{C}{0,1}) = (\overset{C'}{2,2})$ $\quad \overrightarrow{A'B'} = (2,0)$ $\quad R' = \{(1,-1); (2,0), (1,3)\}$

$\overrightarrow{A'C'} = (1,3)$

$$Mf(B_c) = Mf(R, R_c) \cdot C(R_c, R) = \left(\begin{array}{cc|c} 2 & 1 & 1 \\ 0 & 3 & -1 \\ \hline 0 & 0 & 1 \end{array}\right) \cdot \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$

b) $f(\overset{A}{2,1}) = (\overset{A'}{1,2})$ $\quad \overrightarrow{AB} = (-3,-2)$ $\quad R = \{(2,1); (-3,-2), (-2,0)\}$

$f(\overset{B}{-1,-1}) = (\overset{B'}{1,1})$ $\quad \overrightarrow{AC} = (-2,0)$

$f(\overset{C}{0,1}) = (\overset{C'}{2,-1})$ $\quad \overrightarrow{A'B'} = (0,-1)$ $\quad R' = \{(1,2); (0,-1), (1,-3)\}$

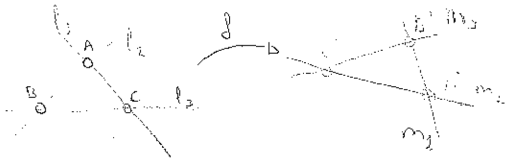
$\overrightarrow{A'C'} = (1,-3)$

$$Mf(B_c) = Mf(R, R_c) \cdot C(R_c, R) = \left(\begin{array}{cc|c} 0 & 1 & 1 \\ -1 & -3 & 2 \\ \hline 0 & 0 & 1 \end{array}\right) \cdot \left(\begin{array}{cc|c} -3 & -2 & 2 \\ -2 & 0 & 1 \\ \hline 0 & 0 & 1 \end{array}\right)$$

c) $f(l_1) = m_1$

$f(l_2) = m_2$

$f(l_3) = m_3$



Buscamos los puntos de corte entre las rectas l_1, l_2, l_3 (A, B, C) y entre las rectas m_1, m_2, m_3 (A', B', C') para calcular los sistemas de referencia R y R'

$R = \{A; \overrightarrow{AB}, \overrightarrow{AC}\}$ $\qquad R' = \{A'; \overrightarrow{A'B'}, \overrightarrow{A'C'}\}$

$$Mf(B_c) = Mf(R, R_c) \cdot C(R_c, R) = \left(\begin{array}{cc|c} \overrightarrow{A'B'} & \overrightarrow{A'C'} & A' \\ \hline 0 & 0 & 1 \end{array}\right) \cdot \left(\begin{array}{cc|c} \overrightarrow{AB} & \overrightarrow{AC} & A \\ \hline 0 & 0 & 1 \end{array}\right)^{-1}$$

(22) $R_1 = \{p; e_1, e_2, e_3\}$ $\quad R_2 = \{q; u_1, u_2\}$

$$Mf(R_1, R_2) = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$R_1' = \{p + e_1 + 2e_3; e_3, e_1 + e_2, e_2 + e_3\}$

$R_2' = \{q + 2u_1; u_1 + u_2, -u_2\}$

$f(p) = (3,0)$

$f(e_1) = (1,0)$

$f(e_2) = (0,1)$

$f(e_3) = (0,1)$

Conocemos los datos de R_1' respecto R_1 y de R_2' respecto de R_2, por tanto:

$$Mf(R_1', R_2') = C(R_2, R_2')\, Mf(R_1, R_2)\, C(R_1', R_1) =$$

$$= \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 1 & -1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)^{-1} \cdot \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) \left(\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

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(23) $f : \mathbb{A}^3 \longrightarrow \mathbb{A}^2$ $\qquad \Pi \equiv x - y + 2z = 1$ $\qquad M f(R_c) = \begin{pmatrix} 2 & -1 & 1 & -1 \\ 1 & 1 & 2 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Tomamos $\{P; \bar{v}_1, \bar{v}_2\}$ referencia del plano Π y calculamos $\underbrace{f(P)}_{P'}$, $\underbrace{\tilde{f}(\bar{v}_1)}_{\bar{v}_1'}$, $\underbrace{\tilde{f}(\bar{v}_2)}_{\bar{v}_2'}$ usando la matriz que nos dan

$P = (1, 0, 0) \qquad \bar{v}_1 = (1, 1, 0) \qquad \bar{v}_2 = (0, 2, 1)$

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \left(\begin{array}{ccc|c} 2 & -1 & 1 & -1 \\ 1 & 1 & 2 & 2 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \Longrightarrow \begin{cases} x = 2 - 1 = 1 \\ y = 1 + 2 = 3 \\ z = 0 \end{cases} \qquad P' = (1, 3, 0)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \Longrightarrow \begin{cases} x = 2 - 1 = 1 \\ y = 1 + 1 = 2 \\ z = 1 \end{cases} \qquad \bar{v}_1' = (1, 2, 1)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \Longrightarrow \begin{cases} x = -2 + 1 = -1 \\ y = 2 + 2 = 4 \\ z = 2 \end{cases} \qquad \bar{v}_2' = (-1, 4, 2)$$

Ya tenemos 1 punto y 2 vectores para formar el plano Π' (imagen de Π)

$$\Pi' = P' + \lambda \bar{v}_1' + \mu \bar{v}_2' \Longrightarrow \begin{cases} x' = 1 + \lambda - \mu \\ y' = 3 + 2\lambda + 4\mu \\ z' = \lambda + 2\mu \end{cases} \longrightarrow \left(\begin{array}{cc|c} 1 & -1 & x' - 1 \\ 2 & -4 & y' - 3 \\ 1 & 2 & z' \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & -1 & x' - 1 \\ 0 & -2 & y' - 3 - 2x' + 2 \\ 0 & 3 & z' - x' + 1 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cc|c} 1 & -1 & x' - 1 \\ 0 & -2 & y' - 3 - 2x' + 2 \\ 0 & 0 & z' - x' + 1 + \frac{3y' - 9 - 6x' + 6}{2} \end{array}\right) \longrightarrow z' - x' + 1 + \frac{3y'}{2} - \frac{9}{2} - 3x' + 3 = 0$$

$$\boxed{\Pi' : \quad 4x' - \frac{3y'}{2} - z' - \frac{13}{2} = 0}$$

(24) $f : \mathbb{A}^4 \longrightarrow \mathbb{A}^4$ $\qquad H_{c,2}$ $\qquad f(\overbrace{2, 0, 1, 0}^{P}) = (-1, 0, 1, 1)$

Sabemos que $H_{c,2} \Longleftrightarrow \overrightarrow{Cf(P)} = k\,\overrightarrow{CP}$, es decir $f(P) - C = k(P - C)$

por lo que $f(P) = k \cdot P + (1 - k) \cdot C$

$$\underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}}_{f(P)} = \left(\begin{array}{cccc|c} 2 & 0 & 0 & 0 & -c_1 \\ 0 & 2 & 0 & 0 & -c_2 \\ 0 & 0 & 2 & 0 & -c_3 \\ 0 & 0 & 0 & 2 & -c_4 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right) \underbrace{\begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}}_{P} \longrightarrow \begin{cases} -1 = 4 - c_1 \\ 0 = -c_2 \\ 1 = 2 - c_3 \\ 1 = -c_4 \end{cases} \longrightarrow \begin{cases} c_1 = 5 \\ c_2 = 0 \\ c_3 = 1 \\ c_4 = -1 \end{cases} \qquad C = (5, 0, 1, -1)$$

(the 4×4 block labelled k, the last column labelled $(1-k)\cdot C$)

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(25) $f : \mathbb{A}^3 \to \mathbb{A}^3$ $\quad S : x + 3y + 2z + 5 = 0$ (plano fijo)

$P = (0,0,0) \to Q(-5, 5, -5)$

Escogemos una referencia $R = \{\underset{\in \pi}{A} ; \underbrace{\bar{v}_1, \bar{v}_2}_{\in \pi}, \overrightarrow{AP}\} = \{(-5,0,0); (2,0,-1), (-3,1,0), (5,0,0)\}$

Sabemos que $A, \bar{v}_1$ y $\bar{v}_2$ al pertenecer a π son fijos

$\tilde{f}(\overrightarrow{AP}) = f(P) - f(A) = Q - A = (-5, 6, -4)$

$$Mf(R_c) = Mf(R, R_c) \cdot C(R_c, R) = \left(\begin{array}{ccc|c} 2 & -3 & -5 & -5 \\ 0 & 1 & 6 & 0 \\ -1 & 0 & -4 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \left(\begin{array}{ccc|c} 2 & -3 & 5 & -5 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)^{-1} = \underbrace{\left(\begin{array}{ccc|c} 0 & -3 & 2 & 5 \\ 1 & 4 & 2 & 5 \\ -1 & -3 & -1 & -5 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}_{B}$$

Para comprobar que B es un sesgo demostramos que $\forall P \notin H$, $\overrightarrow{Pf(P)} \in \overrightarrow{H}$, es decir que $f(P) - P \in \overrightarrow{H}$

$$\underbrace{\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}}_{f(P)} = \left(\begin{array}{ccc|c} 0 & -3 & -2 & -5 \\ 1 & 4 & 2 & 5 \\ -1 & -3 & -1 & -5 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \underbrace{\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix}}_{P} \longrightarrow \begin{cases} x = -3y' - 2z' - 5 \\ y = x' + 4y' + 2z' + 5 \\ z = -x' - 3y' - z' - 5 \end{cases}$$

$\overrightarrow{Pf(P)} = f(P) - P = (-3y' - 2z' - 5,\ x' + 4y' + 2z' + 5,\ -x' - 3y' - z' - 5) - (x', y', z') =$

$= (\underbrace{-x' - 3y' - 2z' - 5}_{x},\ \underbrace{x' + 3y' + 2z' + 5}_{y},\ -x' - 3y' - 2z' - 5)$

Necesitamos que $\overrightarrow{Pf(P)}$ pertenezca a la dirección del plano S

$S \equiv x + 3y + 2z = 0 \longrightarrow (-x' + 3y' - 2z' - 5) + 3(x' + 3y' + 2z' + 5) + 2(-x' - 3y' - 2z' - 5) = 0$

(26)
$$Mf(R_c) = \begin{pmatrix} -1 & -1 & -1 & -2 \\ 4 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} -1 & -1 & -1 & -2 \\ 4 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \longrightarrow \begin{cases} x = -x - y - z - 2 \\ y = 4x + 2y + 3z + 4 \\ z = x + y + z + 1 \end{cases} \longrightarrow \begin{cases} 2x + y + z + 2 = 0 \\ 4x + y + 3z + 4 = 0 \\ x + y + 1 = 0 \end{cases}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 2 & 1 & 1 & 2 \\ 4 & 1 & 3 & 4 \\ 1 & 1 & 0 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 2 & 0 & 2 & 2 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \longrightarrow \begin{cases} x + z = 1 \\ -y + z = 0 \end{cases}$$

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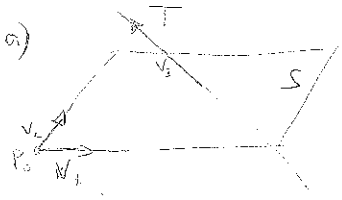
(28) a)



S plano
T recta

Escogemos una referencia $R = \{P_0\,;\ \underbrace{\bar V_1, \bar V_2}_{\in \vec S}, \underset{\in \vec T}{\bar V_3}\}$, $P_0 \in S$

$P_0 = (0,0,1) \qquad \bar V_1 = (1,0,0),\ \bar V_2 = (0,1,0) \qquad V_3 = (0,1,1) \to (0,0,0)$

$$Mf(R_c) = \underbrace{\left(\begin{array}{ccc|c} \bar V_1 & \bar V_2 & \bar 0 & P_0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}_{Mf(R,R_c)} \underbrace{\left(\begin{array}{ccc|c} \bar V_1 & \bar V_2 & \bar V_3 & P_0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)^{-1}}_{C(R_c,R)}$$

b) Proyección sobre T en la direc S: Escogemos una ref $R = \{P_0\,;\ \underset{\in \vec T}{\bar V_1}, \underbrace{\bar V_2, \bar V_3}_{\in \vec S}\}$, $P_0 \in T$

$P_0 = (1,-1,0) \qquad V_1 = (0,1,1) \qquad \bar V_2 = (0,1,0) \to (0,0,0)$
$\bar V_3 = (1,0,0) \to (0,0,0)$

$$Mf(R_c) = \underbrace{\left(\begin{array}{ccc|c} \bar V_1 & \bar 0 & \bar 0 & P_0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}_{Mf(R,R_c)} \underbrace{\left(\begin{array}{ccc|c} \bar V_1 & \bar V_2 & \bar V_3 & P_0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)^{-1}}_{C(R_c,R)}$$

(29) f es simetría afín $\iff f^2 = Id \iff Mf^2 = Id$

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & -1 \\ -1 & 0 & -1 & 1 \\ -2 & -2 & -1 & 2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\left(\begin{array}{ccc|c} 2 & 1 & 1 & -1 \\ -1 & 0 & -1 & 1 \\ -2 & -2 & -1 & 2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{Sí simetría}$$

· Base subespacio de ptos fijos: $f(x) = x$

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \left(\begin{array}{ccc|c} 2 & 1 & 1 & -1 \\ -1 & 0 & -1 & 1 \\ -2 & -2 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right)\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \to \begin{cases} x = 2x + y + z - 1 \\ y = -x - z + 1 \\ z = -2x - 2y - z + 2 \end{cases} \to \begin{cases} x + y + z = 1 \\ x + y + z = 1 \\ 2x + 2y + 2z = 2 \end{cases} \to \boxed{x + y + z = 1}$$

· Direcc. $\tilde f(\bar d) = -\bar d$

$$\begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ -1 & 0 & -1 \\ -2 & -2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{cases} -x = 2x + y + z \\ -y = -x - z \\ -z = -2x - 2y - z \end{cases} \to \begin{cases} 3x + y + z = 0 \\ x - y + z = 0 \\ 2x + 2y = 0 \end{cases} \xrightarrow{x=\lambda} \begin{cases} x = \lambda \\ y = -\lambda \\ z = -2\lambda \end{cases} \quad \bar d = (1,-1,-2)$$

(30) f es una proy. paralela $\Longleftrightarrow f^2 = f$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Longrightarrow \text{Sí proy. paralela}$$

· <u>Base</u>: ptos fijos: $f(\vec{x}) = \vec{x}$

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \longrightarrow \begin{cases} x = x \\ y = x - 1 \\ z = x - y + z - 1 \end{cases} \longrightarrow \begin{cases} x - y = 1 \\ x - y = 1 \end{cases} \longrightarrow \pi \ \boxed{x - y = 1}$$